Polynomials 1

22 December 2023 18:30

Algebraic Manipulation

Q> What is the product of the real roots of the equation

$$x^2+18x+30 = 2\sqrt{x^2+18x+45}$$

Ans:- $y = \sqrt{x^2+18x+45} \longrightarrow y \geq 0$

$y^2 = x^2+18x+30+15 = 2y+15$

$\Rightarrow y^2-2y-15 = 0 \Rightarrow (y-5)(y+3) = 0$

$\longrightarrow$ So $y = -3$ is not possible

$\Rightarrow y = 5$

$\Rightarrow \sqrt{x^2+18x+45} = 5$

$\Rightarrow x^2+18x+45 = 25$

$\Rightarrow x^2+18x+20 = 0$

$\longrightarrow$ So product is 20.

$$ax^2+bx+c = a(x-\alpha)(y-\beta) = 0$$

Then $\alpha+\beta = \dfrac{-b}{a}$, $\alpha\beta = \dfrac{c}{a}$

Q> Solve the system of equations

$$2x_1+x_2+x_3 = 6$$
$$x_1+2x_2+x_3 = 8$$
$$x_1+x_2+2x_3 = 4$$

Ans:- $3(x_1+x_2+x_3) = 18 \Rightarrow x_1 = 0,\ x_2 = 2,\ x_3 = -2$

$x_1+x_2+x_3 = 6$

Q> If $x+y = xy = 3$, find x^3+y^3, find x, y.

Ans:- $x^3+y^3 = (x+y)(x^2-xy+y^2) = (x+y)\left((x+y)^2-3xy\right) = 0$

$(x+y)^2 = x^2+2xy+y^2 = 9 \qquad \Rightarrow x+y = 3$

$(x-y)^2 = x^2-2xy+y^2 = -3 \qquad \Rightarrow x-y = \pm\sqrt{3}\, i$

$\Rightarrow (x,y) = \left(\frac{3+i\sqrt{3}}{2}, \frac{3-i\sqrt{3}}{2}\right), \left(\frac{3-i\sqrt{3}}{2}, \frac{3+i\sqrt{3}}{2}\right)$

Q> Factor x^4+4 into two polynomials of real coefficient

Ans:- $x^4+4 = (x^2+2x+2)(x^2-2x+2)$

Q> Simplify

$(\sqrt{5}+\sqrt{6}+\sqrt{7})(\sqrt{5}+\sqrt{6}-\sqrt{7})(\sqrt{5}-\sqrt{6}+\sqrt{7})(-\sqrt{5}+\sqrt{6}+\sqrt{7})$

Ans:- $\left((\sqrt{5}+\sqrt{6})^2-(\sqrt{7})^2\right)$ $\qquad$ $\left((\sqrt{7})^2-(\sqrt{5}-\sqrt{6})^2\right)$

$= 5+6+2\sqrt{30}-7$ $\qquad$ $= 7-(5+6-2\sqrt{30})$

$= 4+2\sqrt{30}$ $\qquad$ $= -4+2\sqrt{30}$

$(2\sqrt{30})^2-4^2$

$= 4\times 30-16 = 104$

HomeWork

Q> Show that (without multiplying it out),

$$\frac{b-c}{a} + \frac{c-a}{b} + \frac{a-b}{c} = \frac{(a-b)(b-c)(a-c)}{abc}$$

Ans:- Hint: Define polynomials based on variables a, b, c

Ans'— Hint: Define polynomials based on variables a, b, c

HomeWork

Q) Find all solutions $(x_1, x_2, x_3, x_4, x_5)$ of the system of inequalities

$$(x_1^2 - x_3x_5)(x_2^2 - x_3x_5) \leq 0$$
$$(x_2^2 - x_4x_1)(x_3^2 - x_4x_1) \leq 0$$
$$(x_3^2 - x_5x_2)(x_4^2 - x_5x_2) \leq 0$$
$$(x_4^2 - x_1x_3)(x_5^2 - x_1x_3) \leq 0$$
$$(x_5^2 - x_2x_4)(x_1^2 - x_2x_4) \leq 0$$

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = 0$$

↳ Let the roots be $\alpha_1, \alpha_2, \ldots, \alpha_n$

Then,

$$\sum \alpha_i = \frac{-a_{n-1}}{a_n}$$

$$\sum \alpha_i\alpha_j = \frac{a_{n-2}}{a_n}$$

$$\sum \alpha_i\alpha_j\alpha_k = \frac{-a_{n-3}}{a_n}$$

$$\vdots$$

$$\alpha_1\alpha_2\cdots\alpha_n = \frac{(-1)^n a_0}{a_n}$$